A Multiple-Intelligences Approach to Teaching Number Systems

Katrin Becker
Computer Science Education Group
Department of Computer Science
University of Calgary
2400 University Drive NW
Calgary, Alberta Canada
T2N 1N4
(403)220-5769
email: becker@cpsc.ucalgary.ca

Abstract

Gardener's Theory of Multiple Intelligences [Gardner 1997] has implications for how we can develop lessons to cover various concepts and topics. The concept of number systems is presented, framed within the context of multiple intelligences, and various approaches to teaching this lesson are described. The underlying implication is that students are likely to learn "better" when content is presented in more than one format. The Theory of Multiple Intelligences provides us with a convenient framework within which to create these various approaches.

1.0 Introduction

Knowledge of machine representation of data is fundamental to CS. Since all information represented by computer is ultimately represented in binary, an understanding of binary number systems is crucial, and almost all CS programs include this topic somewhere in their elementary curriculum. Often this topic is treated in a rather matter-of-fact manner. Instructors view the material as elementary, and many students view the material as a tedious, necessary evil. It is possible to use the Theory of Multiple Intelligences first developed by Gardner [Gardner 1997] as a basis for inspiration that will allow instructors to vary the approach used, and perhaps even make it interesting.

Current research in education [Taylor, Gilmer, and Tobin, Eds., 2002][Bransford, 2000] suggests that students are better served in the long run when instructors teach with the goal of understanding rather than simple retention. Understanding is more difficult to achieve than remembering. [Wiggins & McTighe 1998] If the primary goal is that learners *understand* number systems, then we must determine how this understanding can be demonstrated. In this case, understanding will be demonstrated as follows. Having mastered the number systems lesson, learners will be able to:

- convert numbers between arbitrary bases [to & from base 10].
- explain an arbitrary base (such as base 5 or 13) without first having been shown that base.
- show / tell / demonstrate conversion of specific numbers from base X to base Y.
- count in an arbitrary base.
- perform simple arithmetic in an arbitrary base.

2.0 Multiple Intelligences & Lesson Design

The Theory of Multiple Intelligences was first proposed by Gardner [Gardner 1997] . As a theory of mind, it claims that intelligence can take multiple forms and that most people excel in just a few, rather than equally in all. Gardner describes eight "intelligences": Linguistic (which deals with words & language); Logical-Mathematical (having to do with numbers, logic); Spatial (visual aspects, such as pictures, charts, and 3D); Musical (music, song, sound); Bodily-Kinesthetic (physical activity); Interpersonal (social awareness & interaction); Intrapersonal (self, introspective, philosophy); and most recently described: Naturalistic (dealing with nature; life sciences).

By acknowledging this theory, we open the possibility for development of various forms of approach to a lesson, which at the very least provides for variety. Given that we accept that this theory has at least some merit, a few assertions can be postulated:

1. All learners can learn to some extent with each (or almost any) approach. How effectively or efficiently they are able to learn using an approach that does not match their strength remains to be shown.
2. It is not possible to fully "understand" something (depth) without involving more than one "intelligence". For example, being able to recite some explanation does not prove mastery. If on the other hand, the learner can demonstrate that their understanding can be transferred from one form of expression to another, it would seem reasonable to assume that they have achieved at least some level of mastery.
3. Thorough assessment (of understanding) is not possible if it is based on a single intelligence. If information can only be repeated in the same context as it was presented, the information may be inert, and not transferable [Bransford, 1989]. Such information is not particularly useful.
4. Most lessons are not "pure" in that they already address more than one intelligence. Many lessons are presented verbally or augmented with verbal or written explanation, even if the kernel of the lesson is visual, or mathematical.

The implications this has for instructional design is that if we present material that has been explicitly framed within the context of this theory, we are likely to reach more learners than we would if we simply used well-known, standard approaches. With a bit of effort, most topics can indeed be framed within the context of this theory.

If we further divide a typical "lesson" into three main components, or aspects, we can use this as a template for new approaches that might not otherwise have presented themselves. The three aspects of a typical lesson are: Attention, where the instructor attempts to engage the class and garner interest; the Activity itself, which is at the heart of the lesson; and Assessment, how we determine if the lesson has been learned. These components serve as a convenient template, but it must be remembered that many aspects of a lesson are also not pure: attention-getting can help learning; activities can gain attention or be used to assess; people can learn from assessments.

Current thinking in Education would dictate that the goals of a lesson and its assessment are intimately linked [Bransford, 2000]. That is, a clear specification of the objectives of a particular teaching exercise would also suggest particular assessment measures. Given that is the case, then, we must answer the question, "Why teach people about Number Systems?"

3.0 Why Teach Number Systems

The ACM/IEEE Computing Curricula 2001 [ACM CC2001] lists low-level representation as being core to a CS curriculum. If we acknowledge the validity of this curriculum document, then this in itself is reason enough. A further rationale is possible.

ACM/IEEE CC2001 Topics

AR2. Machine level representation of data [core hours: 3]

- Bits, bytes, and words
- Numeric data representation and number bases
- Fixed- and floating-point systems
- Signed and twos-complement representations
- Representation of nonnumeric data (character codes, graphical data)
- Representation of records and arrays

Learning objectives:

- Explain the reasons for using different formats to represent numerical data.
- Explain how negative integers are stored in sign-magnitude and twos-complement representation.
- Convert numerical data from one format to another.
- Discuss how fixed-length number representations affect accuracy and precision.
- Describe the internal representation of nonnumeric data.
- Describe the internal representation of characters, strings, records, and arrays.

The rationale behind teaching this unit is fairly clear. The fundamental data form in CS is binary strings and everything else is built on this. An understanding of binary representation helps to understand many other concepts related to numbers. Also, number systems are a higher-level concept from binary or octal. It would stand to reason that if students can be made to understand the higher-level concept, learning any specific number system (like binary, octal, or hex) is simplified. Finally, a deep understanding of abstraction and symbolism is at the very heart of CS and algorithm design. Familiarity of number systems and how they relate to each other as well as how they can be used to represent other things is closely tied to this.

4.0 Openers – Getting Attention

The "opener" is typically the beginning of a lesson, lecture, or unit that is designed to capture the learner's attention and inspire them to pay attention to the rest. The "opener" often consists of the instructor stating the topic ("Today we're going to talk about ..."). The Theory of Multiple Intelligences gives us a basis from which to vary this approach.

A linguistic approach to an opener might involve a story, for instance: *"Aliens have landed and are starting to ask questions. They want to know about this <u>METRIC</u> thing."* The idea here is that the rest of the lesson can then be framed within the context of attempting to explain our counting system to aliens who are completely unfamiliar with it. Sometimes an opener can be as simple as a question like: *"Why do we count using base 10?"* This would count as a logical-mathematical approach. This question could be used to begin a discussion around the abstract nature of our current counting system [Bucci 2001]. There is, in fact no natural basis (aside from the number of fingers we possess) for using base 10.

Some questions can be posed as queries to ponder: *"What do you suppose would be different in the world if we only had 8 fingers?"* (Logical-Mathematical, Interpersonal/Social). An opener that takes advantage of spatial intelligence would be *"By the time we are done today, you'll know how to count to 1000 on your fingers."* [Selvia, 1995]

If the facilities permit it, a musical approach could include playing the song "New Math" by Tom Lehrer [Lehrer 1965]. This song, written by a Mathematician runs through a subtraction problem first in base 10, and then in base 8.

For some learners, an intrapersonal (somewhat philosophical) approach works best. This would involve explaining to class why learning about number systems is useful. This approach is one of the ones that is almost always helpful, at least to some extent.

While dancing around the lecture theatre is an uncommon activity in a CS class, there are still a number of physical activities that can be acted out to address the needs of those learners who have a strong bent towards learning through Bodily-Kinesthetic Intelligence. Get the class to fold a piece of paper in half, then in half again, then in half again,... until they can't any more. This provides a means of introducing the concept of binary numbers.

Perhaps one of the most difficult facets to address in a CS course is the naturalistic one. Often when struggling for a natural analogy of some concept, we end up in mathematics, which may seem very natural for experts, but is often highly un-natural to novices. A common example of this would be the use of Fibonacci numbers to demonstrate recursion. It has been shown that connecting a new concept to something the learner is already familiar with helps them to construct new meaning and make new connections [Mintzes 1997]. One of the problems with the Fibonacci numbers example of recursion is that we often end up having to introduce a new concept in order to introduce a new concept. There may be a certain poetic irony in this approach, but it rarely helps the students understand. Perhaps a more appropriate opener that engages the naturalistic intelligence might be to explain the "6 Degrees of Separation" Theory. [Guare, 1990]

5.0 The Core of the "Lesson"

Many instructors teach number systems using primarily mathematical approaches. While there is nothing wrong with this, the author is suggesting alternate approaches that can be used to augment, rather than replace them. In fact, the mathematical approach is ultimately how students must be able to work with various number bases and representations, and as such can be considered foundational. However, it may not be the most "user-friendly" approach to take. Many novice students are still somewhat fearful of mathematics and introducing this topic from a purely mathematical perspective may leave many students lost precisely when they should be encouraged and inspired.

The following brief sections explain various approaches to teaching about number systems, some well known and quite traditional, others novel. Each is identified as to which "intelligences" it engages.

5.1 Explain how numbers are built [Engages: Logical-Mathematical, Linguistic]

One approach is to start by explaining base 10, as that is the one base we can assume everyone is already familiar with. This typically feels rather awkward, both for the instructor and the class as the decimal system is so familiar that we have long since stopped examining it in this way. This is where the attention-getting technique of trying to explain our number system to aliens can help us frame this exercise. Describing the decimal number system allows us to establish the terminology and approach we are using. Having done that, we can then use an identical approach to explain base 8, then base 2, and then base 16. Approaching the explanation in this order gives us the ability to isolate the "variables" and point out the pattern. The initial description of decimal numbers draws on principles and concepts learned in primary school. Decimal numbers are:

- Represented by 10 distinct symbols: 0,1,2,3,4,5,6,7,8,9
- Based on powers of 10
- Each place to the left of a digit in a string increases by a power of 10; each place to the right of a digit in a string decreases by a power of 10

Example: 4769210 in expanded notation looks like:

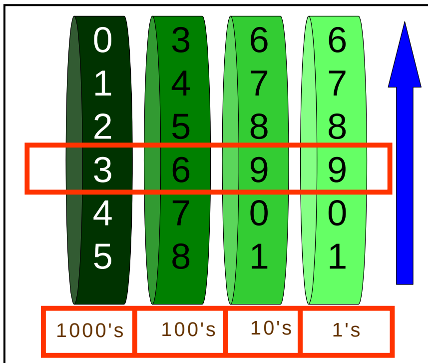
= 4 * 104 + 7 * 103 + 6 * 102 + 9 * 101 + 2 * 100

= 4 * 10000 + 7 * 1000 + 6 * 100 * 9 * 10 + 2 * 1

Once the idea of describing numbers in this way has been introduced, we can expand on this by generalizing the rules. The general rules that follow have been described in a way that allows them to be applied to any arbitrary number base.

General Rules:

- x0 = 1; x1 = x; x2 = x * x; x-1 = 1/x; x-2 = 1/ (x*x);
- Leading zeros are not significant, and unless they appear to the right of a decimal place have no effect on the value of the number.
- When adding and subtracting the decimal points of real numbers must be vertically aligned.
- When dividing two real numbers they must both be adjusted (multiplied by their base) until the divisor is an integer.
- For real number addition and subtraction the exponents must be the same.
- For real number multiplication one must multiply the mantissas and add the exponents.
- For real number division one must divide the mantissas and subtract the exponents.

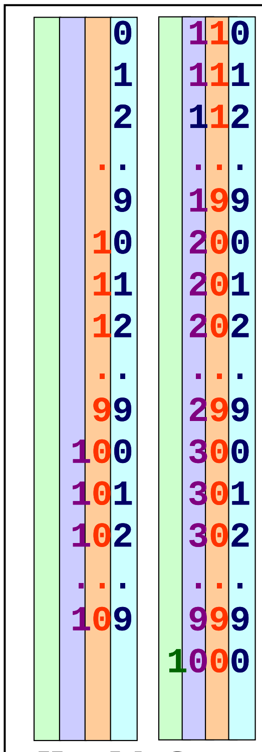


Odometer

This approach integrates three forms of intelligence, namely: linguistic, logical, and spatial. Some people are weaker in one or more of these areas, and as a result some students have difficulty following this explanation because of the integration required. Augmenting this approach with others that engage other forms of intelligence are likely to facilitate retention and transfer. Someone who is very visual for example, may feel more comfortable with the next approach. Having acquired a "sense" of the concept, they will then be able to return to the more formal approach and try it again.

5.2 The Odometer Analogy [Engages: Spatial; Bodily-Kinesthetic]

Another approach that draws on spatial abilities is to describe and show how number systems and counting can be represented using an odometer. [Heller, S., 1997] Again, it is recommended that the explanation begins with base 10 as this is the form of odometer with which most students will have prior experience. A different base can be represented by simply changing the "size" of the wheels in the odometer. If we make the wheel a bit smaller, we only have room for 8 numbers, and we end up with octal. The mechanism remains unchanged. Binary can also be done, using a very small "wheel". As a diversion from the usual exercises, students can actually build a paper version of an odometer and use it to explore counting. One great advantage of this approach is that it allows students to see first hand how two's compliment arithmetic operates. We can count both forwards and backwards. It also allows us to look at representational limits, over- and underflow, and of we take it far enough we can also use it to begin to explain some of the problems encountered when working with floating point numbers. Although they are hard to find these days, old mechanical adding machines operate in much the same fashion and make excellent demonstration tools.



How We Count

5.3 Look at how we count (then do the same in other bases). [Engages: Logical-Mathematical; Linguistic; Spatial (patterns)]

An alternative to the odometer approach examines "counting". [Petzold, C., 1999] We look at how numbers grow in size and magnitude and then apply the same technique using other bases. Once we have demonstrated several other bases, it becomes possible to identify similar elements and thereby establish the pattern used.

Division	Quotient	Remainder	Binary Number
2671 / 2	1335	1	1
1335 / 2	667	1	11
667 / 2	333	1	111
333 / 2	166	1	1111
166 / 2	83	0	0 1111
83 / 2	41	1	10 1111
41 / 2	20	1	110 1111
20 / 2	10	0	0110 1111
10 / 2	5	0	0 0110 1111
5 / 2	2	1	10 0110 1111
2 / 2	1	0	010 0110 1111
1 / 2	0	1	1010 0110 1111

Converting to Base 10

5.4 Show How to Convert Numbers from Some Base to Base 10. [Engages: Logical-Mathematical]

The ability to convert numbers from base 10 to some other base as well as from some arbitrary base back to base 10 is often required when working with low-level representations. It is also sometimes helpful when deciphering certain classes of program errors. As in the previous approaches, starting with decimal sets the "scene".

Example: 101110012 in *expanded notation* looks like:

= 1 * 27 + 0 * 26 + 1 * 25 + 1 * 24 + 1 * 23 + 0 * 22 + 0 * 21 + 1 * 20

= 1 * 128 + 0 * 64 + 1 * 32 + 1 * 16 + 1 * 8 + 0 * 4 + 0 * 2 + 1 * 1

= 128 + 32 + 16 + 8 + 1

= 185

5.5 Show how to convert numbers from base 10 to others. [Engages: Logical-Mathematical]

Converting ***from*** some arbitrary base is often viewed by novices as being simpler than the reverse (converting ***to*** some arbitrary base), perhaps because the former involves multiplication and the latter division. As a result these two conversion techniques can be tackled separately, and it is recommended that they be tackled in the order listed.

A Musical Approach

5.6 Relate octal numbers to the musical scale. [Engages: Musical; Spatial (patterns)]

This approach is as yet untried in a classroom environment, although the idea has been presented to various students and other teachers with encouraging reviews. An understanding of octal number systems can be constructed by mapping octal numbers onto the musical scale, and using them to encode various tunes. This exercise would lend itself best to implementation in a small class or lab environment. It could even be adapted as an on-line, interactive exercise.

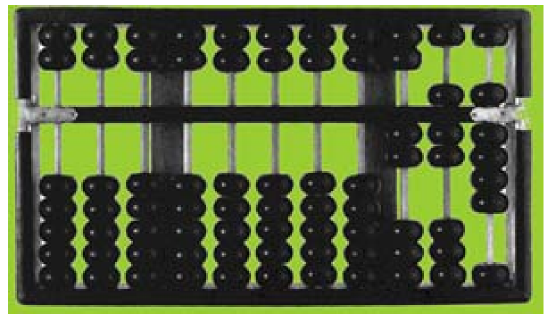
5.7 Show How to Count in Binary on Your Fingers. [Engages: Bodily-Kinesthetic; Spatial]

One of the un-acknowledged strengths of this approach is the kinesthetic nature of this activity.

Binary Finger Counting
Image Credit: [ivan]

The tactile nature of this exercise helps some students internalize the patterns involved in binary counting. They can recall the physical movements, which in turn can trigger their logical recollection of the concept. Also, it's fun (watch out for the number four).

5.8 Use an Abacus [Engages: Bodily-Kinesthetic; Spatial; Intrapersonal (leave them to play with it)]


Abacus

This approach has definite constructive overtones as the exercise will typically involve the learners trying to solve various problems using the abacus with the hope that one side-effect of this activity will be a sort of 'epiphany' involving the relationships of numbers, counting, and arithmetic, that can be applied to various number systems. If resources permit, devices can be built that behave just like an abacus, but that employ various number bases – which allows for comparison.

Act It Out

5.9 Act It Out [Engages: Bodily-Kinesthetic; Interpersonal]

For this activity, each person gets a wheel with numbers on it, a list, or a flip-book of numbers. The 'action' is that we have the participants count out loud, and when one gets to '9' (s)he gets to poke the next guy beside him(her). The original 'counter' reverts to '0' (zero), and the next guy (the one who was poked) remembers the next number. The key element here is that each participant remembers only their own 'behaviours'. The audience gets to see this "counter" in action so the entire class can be involved. In a university or college environment such "performance art" is rare in the sciences, but often leaves a lasting impact.

5.10 Multiplying like Bunnies [Engages: Naturalistic; Spatial (patterns)]

As stated previously, opportunities for a naturalistic approach are sometimes difficult to create in CS. When dealing with number systems, we can relate a number system to generations of bunnies, each having 'N' babies. In this way 'N' can be 2, 8, 10, or even 16.

6.0 Assessment

No teaching unit can rightfully be considered complete if it lacks a means of assessment. The success or viability of any given approach cannot be determined if there is no way to assess the learner's mastery of the subject matter. [Resnick & Resnick 1992] In this aspect as well as the others, it is possible to focus on various intelligences. This gives us a means of determining whether the material the students were to master can be transferred from one arena to another.

Base 10	Base 2	Base 8	Base 16
32			
	11010		
			FF
		647	

A Worksheet

A musical assessment might be to propose a numerical code (octal mapping) for musical notes. Encode a simple song - try reading it using the numerical code. Requiring students to explain some given base 'X' (such as 5 or 13) using symbols and/or powers is another means of determining mastery. An instrument that can be used both for practice and assessment is the well-known "worksheet", where students can fill in the blanks. This makes use of the Logical-Mathematical, as well as the Linguistic aspects of intelligence. Variations would include questions such as, "What's the next number in base 'X'?" or simple addition exercises in various bases. A Naturalistic approach might include a requirement to find examples in nature (such as asexual reproduction or propagation). Bodily-Kinesthetic, Spatial, Interpersonal intelligences can be combined with each other and/or verbal skills with tasks such as, "Show me ***n*** in binary using your hands". The idea of performance art can also be used to effect here by getting individuals to be "bits": standing = 1; sitting = 0. An entire class can be made to do counting or arithmetic using people. While a more light-hearted approach may become tiresome if used to excess, its occasional implementation can be very effective.

7.0 Conclusion

The Theory of Multiple Intelligences is still classified as a theory, but this need not prevent us from using it a tool to help us create a rich and varied learning environment. The fact that the learners we must reach are technically adults should also not prevent us from trying a more light-hearted approach from time to time. When used to augment sound, well-tested approaches the result can be that some students who were traditionally lost by a primarily linguistic or mathematical approach may be able to use these other approaches to form the necessary connections and build the required understandings.

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